EFFICIENCY OF SEMICONDUCTOR THERMOELECTRIC COOLERS AND HEATERS FOR LIQUID AND GAS FLOWS

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Analytical relationships for determining the energy characteristics of thermopiles used for cooling or heating liquid flows are obtained. The method of calculating the optimum parameters of batteries is discussed.

Semiconductor heat pumps for use in conditioners, thermostats, evaporators, and other devices are now being developed. The efficiency of heat pumps is greatest when there is a continuous flow of the liquid or gas to be heated or cooled. The existing method of calculating the parameters of thermoelectric devices with specified junction temperatures [1–3] or temperatures of the heating or cooling medium [4, 5] applies to cases where all the elements of the thermopile are in the same temperature regime. The efficiency of the whole device can then be evaluated by analyzing the operating conditions of one thermoelement.

A characteristic feature of coolers and heaters for liquid flows is that the heat regime of the individual thermoelements is different. The temperature of the elements varies continuously along the direction of the liquid flow over the cold and hot junctions. A method of determining the optimum parameters of such devices has not yet been devised. This paper derives some relationships for determining the energy and temperature characteristics of semiconductor thermoelectric devices in which the liquid flows successively over the thermopile elements. Various conditions are considered. Such devices are essentially recuperative heat exchangers in which Peltier heat is absorbed and released on the surface of the wall separating the flows of liquid and Joule heat is released within the wall. In the considered devices the heat-transfer media can move relative to one another in different ways.

We will analyze the main types of parallel flow, cocurrent and countercurrent.

We will determine the change in the temperature of the liquid flowing over the hot and cold junctions of the thermopile. We assume that the width of the individual elements is small in comparison with the total length of the thermopile, and then the change in temperature along the x axis can be regarded as continuous. In steady-state conditions heat transfer between the thermoelements and the liquid flow is characterized by the equations

$$W_1 \frac{dT_1}{dx} = a_1 p (T_1' - T_1),$$
$$\pm W_2 \frac{dT_2}{dx} = a_2 p (T_2' - T_2). \qquad (1)$$

The plus sign corresponds to cocurrent flow and the minus sign to countercurrent flow.

The heat balance on the thermopile junctions in the steady state can be written [1] in the form

$$a_1 (T_1 - T_1') = \alpha j T_1' - \frac{1}{2} j^2 \rho d - \frac{\lambda}{d} (T_2' - T_1'),$$
$$a_2 (T_2' - T_2) = \alpha j T_2' + \frac{1}{2} j^2 \rho d - \frac{\lambda}{d} (T_2' - T_1'). \qquad (2)$$

Equations (1) and (2) are valid when the conductive heat fluxes in the axial direction in the liquid and in the thermopile are negligibly small. The validity of neglecting axial heat fluxes in heat exchangers is confirmed by experiments and analytical estimates [6]. In addition, the heat transfer coefficients are assumed constant along each of the surfaces of the thermopile, while the parameters α, ρ, and λ of the thermoelements and the specific heat of the liquid are assumed to be independent of the temperature.

We reduce Eqs. (1) and (2) to the dimensionless form

$$\frac{d\tau_1}{d\xi} = N_1 (\tau_1' - \tau_1),$$
$$\pm \frac{d\tau_2}{d\xi} = N_2 (\tau_2' - \tau_2), \qquad (3)$$
$$\mathrm{Bi}_1 (\tau_1 - \tau_1') = \nu\tau_1' - \frac{1}{2} \nu^2 - (\tau_2' - \tau_1'),$$
$$\mathrm{Bi}_2 (\tau_2' - \tau_2) = \nu\tau_2' + \frac{1}{2} \nu^2 - (\tau_2' - \tau_1'). \qquad (4)$$

In the case of cocurrent flow the temperatures of the liquid flow at x = 0 $T_1|_{x=0} = T_1^{(0)}$, $T_2|_{x=0} = T_2^{(0)}$ or, in dimensionless form, $\tau_1|_{\xi=0} = \tau_1^{(0)}$, $\tau_2|_{\xi=0} = \tau_2^{(0)}$. In the case of countercurrent flow we have $T_1|_{x=0} = T_1^{(0)}$, $T_2|_{x=l} = T_2^{(0)}$ or $\tau_1|_{\xi=0} = \tau_1^{(0)}$, $\tau_2|_{\xi=1} = \tau_2^{(0)}$.

The solution of system (3), (4) with the indicated boundary conditions gives the following distribution of temperature τ_1 and τ_2 in the flows of liquid as functions of the coordinate ξ:

for cocurrent flow

$$\tau_1(\xi) = \frac{1}{\psi} [\tau^{(1)} (\psi \operatorname{ch} \psi\xi - v \operatorname{sh} \psi\xi) + \tau^{(2)} G_1 \operatorname{sh} \psi\xi] \times$$
$$\times \exp(-u\xi) - 1 + \frac{\nu}{2}, \qquad (5)$$

$$\tau_2(\xi) = \frac{1}{\psi} [\tau^{(2)} (v \operatorname{sh} \psi\xi + \psi \operatorname{ch} \psi\xi) +$$
$$+ \tau^{(1)} G_2 \operatorname{sh} \psi\xi] \exp(-u\xi) - 1 - \frac{\nu}{2}, \qquad (6)$$

the temperature drop in the flow of the liquid undergoing cooling being

$$\Delta\tau_1 = \tau_1|_{\xi=0} - \tau_1|_{\xi=1} =$$
$$= \tau^{(1)}\left[1 - \left(\operatorname{ch}\psi - \frac{v}{\psi}\operatorname{sh}\psi\right)\exp(-u)\right] -$$
$$- \tau^{(2)}\frac{G_1}{\psi}\exp(-u)\operatorname{sh}\psi, \qquad (7)$$

the temperature increase in the flow of liquid being heated being

$$\Delta\tau_2 = \tau_2|_{\xi=1} - \tau_2|_{\xi=0} =$$
$$= \tau^{(2)}\left[\left(\frac{v}{\psi}\operatorname{sh}\psi + \operatorname{ch}\psi\right)\exp(-u) - 1\right] +$$
$$+ \tau^{(1)}\frac{G_2}{\psi}\exp(-u)\operatorname{sh}\psi, \qquad (8)$$

for countercurrent flow,

$$\tau_1(\xi) =$$
$$= \frac{\tau^{(1)}[u\operatorname{sh}\varphi(1-\xi) + \varphi\operatorname{ch}\varphi(1-\xi)] + \tau^{(2)}G_1\exp v\operatorname{sh}\varphi\xi}{(u\operatorname{sh}\varphi + \varphi\operatorname{ch}\varphi)\exp v\xi} -$$
$$- 1 + \frac{\nu}{2}, \qquad (9)$$

$$\tau_2(\xi) = \frac{\tau^{(2)}[u\operatorname{sh}\varphi\xi + \varphi\operatorname{ch}\varphi\xi]\exp v + \tau^{(1)}G_2\operatorname{sh}\varphi(1-\xi)}{(u\operatorname{sh}\varphi + \varphi\operatorname{ch}\varphi)\exp v\xi} -$$
$$- 1 - \frac{\nu}{2}, \qquad (10)$$

$$\Delta\tau_1 = \tau_1|_{\xi=0} - \tau_1|_{\xi=1} = \tau^{(1)}\left[1 - \frac{\varphi\exp(-v)}{u\operatorname{sh}\varphi + \varphi\operatorname{ch}\varphi}\right] -$$
$$- \tau^{(2)}\frac{G_1\operatorname{sh}\varphi}{u\operatorname{sh}\varphi + \varphi\operatorname{ch}\varphi}, \qquad (11)$$

$$\Delta\tau_2 = \tau_2|_{\xi=0} - \tau_2|_{\xi=1} = \tau^{(2)}\left[\frac{\varphi\exp v}{u\operatorname{sh}\varphi + \varphi\operatorname{ch}\varphi} - 1\right] +$$
$$+ \tau^{(1)}\frac{G_2\operatorname{sh}\varphi}{u\operatorname{sh}\varphi + \varphi\operatorname{ch}\varphi}. \qquad (12)$$

In formulas (5)–(12)

$$\tau^{(1)} = \tau_1^{(0)} + 1 - \nu/2,\ \tau^{(2)} = \tau_2^{(0)} + 1 + \nu/2;$$
$$G_{1,2} = R_{1,2}[1 + \beta_1 + \beta_2 + \nu(\beta_1 - \beta_2 - \nu\beta_1\beta_2)]^{-1};$$
$$u = \frac{1}{2}[G_1(1 + \nu - \beta_2\nu^2) + G_2(1 - \nu - \beta_1\nu^2)],$$
$$v = \frac{1}{2}[G_1(1 + \nu - \beta_2\nu^2) - G_2(1 - \nu - \beta_1\nu^2)];$$
$$\psi = (v^2 + G_1G_2)^{1/2},\quad \varphi = (u^2 - G_1G_2)^{1/2}.$$

It is easy to see that in the absence of a current in the thermopile ($\nu = 0$) formulas (5)–(12) are converted to the known expressions for the temperature of heat transfer media in recuperative heat exchangers.

It is of interest to note that in the case of countercurrent flow the value of φ may be imaginary for a particular ratio of the parameters, i.e., the temperature of the thermoelement junctions and heat-transfer media will vary along the thermopile in accordance with a sine law with an exponentially varying amplitude. The period of the temperature wave in this case is $2\pi(l/\varphi)^i$ [sic]. A periodic distribution of temperature along the thermopile occurs when the current density lies between ν_1 and ν_2, and

$$\nu_{1,2} = \frac{1}{2}\{W_2 - W_1 + [(W_2 - W_1)^2 +$$
$$+ 4(\beta_1 W_1 + \beta_2 W_2)(\sqrt{W_2} \pm \sqrt{W_1})^2]^{1/2}\}(\beta_1 W_1 + \beta_2 W_2)^{-1}. \qquad (13)$$

The creation of a stable periodical temperature field with a finite thermal resistance on the junctions is due to the action of opposing factors, Peltier heat and the heat flux through the branches of the thermoelement. The relationship between these factors varies along the thermopile.

In addition to the values of the temperature drop the main parameters characterizing the considered devices are the cooling capacity $Q_1 = W_1\Delta T_1$, and the heating capacity $Q_2 = W_2\Delta T_2$, and also the coefficients of performance ε and η:

$$1/\varepsilon = Q_2/Q_1 - 1 =$$
$$= W_2\Delta T_2/W_1\Delta T_1 - 1 = R_1\Delta\tau_2/R_2\Delta\tau_1 - 1, \qquad (14)$$
$$1/\eta = 1 - Q_1/Q_2 =$$
$$= 1 - W_1\Delta T_1/W_2\Delta T_2 = 1 - R_2\Delta\tau_1/R_1\Delta\tau_2. \qquad (15)$$

The expressions characterizing the efficiency of a thermopile [the main ones are the relationships (7), (8), (11), and (12), which determine the values of $\Delta\tau_1$ and $\Delta\tau_2$] are complicated and do not allow an analytical determination of the optimum parameters of the thermoelements and the power supply in general form. For the solution of specific technical problems connected with the construction of thermoelectric devices the values of the required parameters can be found by carrying out a series of numerical calculations on the basis of the formulas obtained above. Computing technique should be used for the calculations. The use of dimensionless similarity criteria reduces the number of variable factors, and the results can be put in general form. Depending on what parameters of the thermopile are prescribed in its design, different criteria R, N, or P can be put into the main formulas for the determination of $\Delta\tau$. The values of these criteria depend on the dimensions of the thermopile, the heat transfer coefficients, and the water equivalents of the flows.

In the preliminary estimate of the parameters of a thermopile it is convenient in some cases to use approximate relationships suitable for the specific design or conditions of application of the thermoelectric device. Such expressions for $\Delta\tau$ and ε can easily be obtained by the appropriate limiting transition, such as for small heat loads ($\nu \ll 1$), for a thermopile of relatively small area ($R_{1,2} \ll 1$), and so on.

Devices in which the water equivalent of the liquid on one side of the battery is much greater than that on the other (air-water conditioners, and so on) are often used. In these cases, when $n = W_1/W_2 \ll 1$ and any relative direction of flows, the expressions for $\Delta\tau$ are converted to the form

$$\Delta\tau_1 = \left\{ \tau_1^{(0)} - \frac{\tau_2^{(0)} + \frac{1}{2}\nu^2[1+(2-\nu)\beta_2]}{1+\nu-\nu^2\beta_2} \right\} \times \times \{1-\exp[-G_1(1+\nu-\nu^2\beta_2)]\}, \quad (16)$$

$$\Delta\tau_2 = n(\Delta\tau_1 + \tau^{(2)}R_1\nu^2)(1+\nu-\nu^2\beta_2)^{-1}. \quad (17)$$

An analysis of the obtained relationships leads to some general conclusions regarding the effect of various factors on the efficiency of a thermopile. The cooling capacity, as in the case of a single element, is greatest at a particular value of current density ν.

The coefficients of performance ε when $\tau_1^{(0)} \geq \tau_2^{(0)}$ decreases steadily with increase in ν from infinity to zero; if $\tau_2^{(0)} < \tau_2^{(0)}$, then ε has a maximum at a particular ν. Countercurrent flow always ensures higher efficiency than cocurrent flow. The values of $\Delta\tau$, ε, and η increase along with the heat transfer coefficients a_1 and a_2. The temperature drop $\Delta\tau_1$ in the case of optimum supply current ν has a maximum when the thermopile has a particular area. With subsequent increase in S in the case of cocurrent flow $\Delta\tau_1$ steadily decreases and in the case of countercurrent flow it tends to a constant limit. The limiting degree of cooling of the flow, determined by the physical properties of the thermoelements (parameter z), is attained with a constant temperature on the hot side of the thermopile (W_2, $a_2 \to \infty$) and area $S \to \infty$.

In the design of a specific semiconductor cooler or heater the required values of ΔT (or Q), W_1, W_2, and the characteristics of the thermoelement material are usually prescribed. The values of a_1 and a_2 are usually also known beforehand. If these data are given, we are faced with the problem of choosing the dimensions of the thermopile to ensure the obtention of the prescribed values of $\Delta\tau_1$ or $\Delta\tau_2$ for maximum ε or η. Another formulation of the problem requires the determination of the parameters of the thermopile to ensure maximum cooling capacity in particular conditions.

If none of the dimensions of the thermopile is prescribed, there are no optimum geometric parameters. The energy efficiency increases steadily with simultaneous increase in S and d. The following typical cases of restrictions on the dimensions of the thermopile can be mentioned.

1. Design considerations prescribe the area of the thermopile (criterion N), and the initial temperatures of the flows, flow rate of the liquid, and heat transfer coefficients are also known. It is required to find the height of the thermoelements (or the value of the Biot number) to secure maximum ε for a prescribed cooling capacity. The appropriate calculation can be carried out using relationships (7) and (8), or (11) and (12). As an example Fig. 1a shows the family of curves $\varepsilon = f(1/Bi_1)$ for different values of $\Delta\tau_1$ (cocurrent flow). The calculations were performed by a BESM-2 electronic digital computer. The upper branch of these curves corresponds to the ascending part of the relationship $\Delta\tau_1(\nu)$ and the lower corresponds to the descending part. Only the upper parts are of practical value.

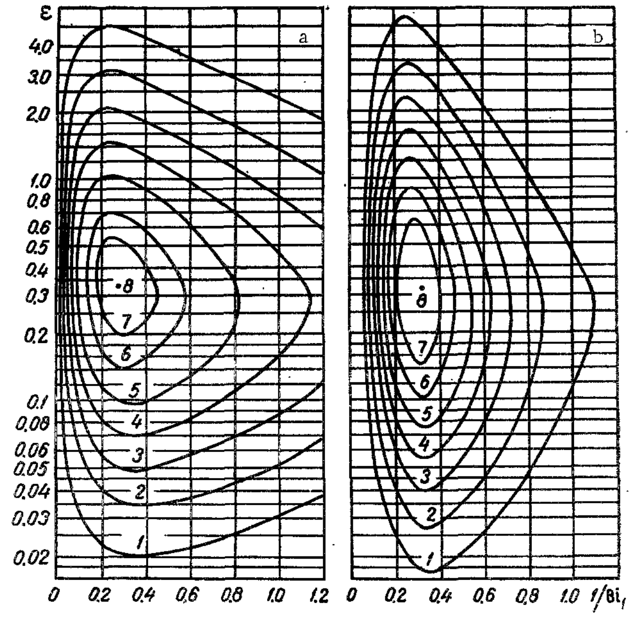

Fig. 1. Coefficient of performance ε as a function of Biot number (height of thermoelements) for a prescribed area (N = 3) (a) or volume of thermopile (P_1 = = 15) (b) and various temperature drops in the flow (cocurrent flow; $m = a_1/a_2 = 0.5$; $n = W_1/W_2 = 0.25$; $\tau_1^{(0)} = \tau_2^{(0)} = 0.6$): 1) $\Delta\tau_1 = 0.01$; 2) 0.015; 3) 0.02; 4) 0.025; 5) 0.03; 6) 0.035; 7) 0.038 (a) and 0.041 (b); 8) 0.0401 (a) and 0.0428 (b).

Curves similar to those in Fig. 1 are obtained for countercurrent flow. For the same initial data the values of ε at the maximum for countercurrent flow are approximately 10% higher than in the case of cocurrent flow. The greatest attainable value of $\Delta\tau_1$ is $4.24 \cdot 10^{-2}$, as against $4.01 \cdot 10^{-2}$ for cocurrent flow.

Using the approximate expression for ε for small $\Delta\tau$ (the first terms of the expansion in ν) we can analytically find the optimum values $\beta_1 = \beta_1^*$ from the condition $d\varepsilon/d\beta_1 = 0$. For cocurrent and countercurrent flow for $\tau_1^{(0)} = \tau_2^{(0)}$ we obtain transcendental equations to determine β_1^*:

$$[1+\beta_1(1+m)]\left\{\frac{2u_0}{1-\exp(-u_0)} - [1+\beta_1(1+m)]\right\} = \frac{\tau^{(0)}}{\tau^{(0)}+1}, \quad (18)$$

$$(\exp v_0 - n)[1+\beta_1(1+m)] \times \left\{\frac{2v_0}{\exp v_0 - 1} - [1+\beta_1(1+m)]\frac{1-n\exp(-v_0)}{1-n}\right\} = \frac{(1-n)\tau^{(0)}}{\tau^{(0)}+1}; \quad (19)$$

$u_0 = u|_{\nu=0}$, $v_0 = v|_{\nu=0}$ must be used in the form

$$u_0 = N_1\frac{\beta_1(1+n)}{1+\beta_1(1+m)}, \quad v_0 = N_1\frac{\beta_1(1-n)}{1+\beta_1(1+m)}. \quad (20)$$

Although the values of β_1^* obtained from Eqs. (18) and (19) relate to the case $\Delta\tau_1 \to 0$, they can be used for the whole range of variation of $\Delta\tau_1$ since β^* depends weakly on $\Delta\tau_1$.

2. The volume of the thermoelectric material (criterion P) and the thermotechnical parameters of the thermopile are prescribed. It is required to find the optimum ratio of the height of the elements and the area of the thermopile from the expressions for $\Delta\tau_1$ and $\Delta\tau_2$. The results of such a calculation are shown in Fig. 1b.

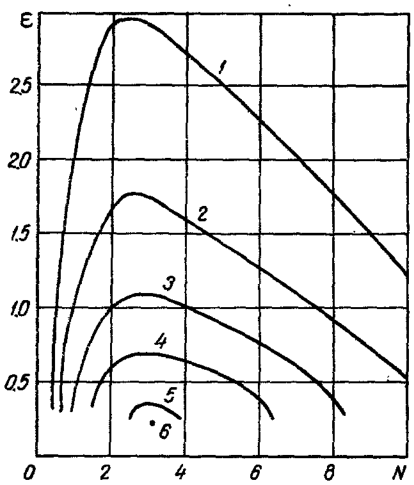

Fig. 2. Coefficient of performance ε as a function of parameter N (area of thermopile) for a prescribed thickness of thermopile and different temperature drops in the flow (cocurrent flow; $Bi_1 = 1.25$; $m = a_1/a_2 = 0.5$; $n = W_1/W_2 = 0.25$; $\tau_1^{(0)} = \tau_2^{(0)} = 0.6$): 1) $\Delta\tau_1 = 0.01$; 2) 0.015; 3) 0.02; 4) 0.025; 5) 0.03; 6) 0.0312.

The abscissae of the maxima on the curves $\varepsilon = f(1/Bi)$ correspond to the optimum values of β_1^*, which determine the height d for the given operating conditions of the thermopile. The value of β_1^* for cocurrent and countercurrent flow at $\tau_1^{(0)} = \tau_2^{(0)}$ and $\Delta\tau_1 \to 0$ can be found also from the equations

$$2[1+\beta_1(1+m)]\times\left[\frac{u_0}{1-\exp(-u_0)}-\frac{1+\beta_1(1+m)}{2+\beta_1(1+m)}\exp u_0\right]=\frac{\tau^{(0)}}{\tau^{(0)}+1}, \quad (21)$$

$$(\exp v_0-n)[1+\beta_1(1+m)]\times\left[\frac{v_0}{\exp v_0-1}-\frac{1+\beta_1(1+m)}{2+\beta_1(1+m)}\times\frac{1-n\exp(-v_0)}{1-n}\right]=\frac{(1-n)\tau^{(0)}}{\tau^{(0)}+1}; \quad (22)$$

where

$$u_0=P_1\frac{\beta_1^2(1+n)}{1+\beta_1(1+m)},\quad v_0=P_1\frac{\beta_1^2(1-n)}{1+\beta_1(1+m)}. \quad (23)$$

3. The thermopile is composed of standard thermoelements of height d. The thermotechnical parameters of the battery are prescribed, and it is required to find its optimum area (criterion N). An example of the data obtained by calculation on an electronic computer is shown in Fig. 2 [upper branches of curves $\varepsilon = f(N)$]. The optimum value $N = N^*$ can also be found from equations similar to those given above (they are obtained for cocurrent and countercurrent flow from the condition $\left.\frac{\partial\varepsilon}{\partial N}\right|_{\Delta\tau_1\to 0}=0$):

$$[1+\beta_1(1+m)]\left[\frac{2u_0}{1-\exp(-u_0)}-\exp u_0\right]=\frac{\tau^{(0)}}{\tau^{(0)}+1}, \quad (24)$$

$$(\exp v_0-n)[1+\beta_1(1+m)]\times\left[\frac{2v_0}{\exp v_0-1}-\frac{n\exp(-v_0)}{1-n}\right]=\frac{\tau^{(0)}}{\tau^{(0)}+1}. \quad (25)$$

The quantities u_0 and v_0 must be taken in the form (20). Formulas (24) and (25) give results which practically coincide with the exact data for the position of the maxima on the curves $\varepsilon = f(N)$ with $\tau_1^{(0)} = \tau_2^{(0)}$ for a wide range of cooling capacities.

NOTATION

T is the temperature in the liquid flow; ΔT is the difference in flow temperatures along the thermopile; $T^{(0)}$ is the temperature of heat transfer medium at entrance; T' is the junction temperature; V, S, d, l, p are the volume, area, thickness, length, and width of the thermopile; α, λ, ρ are the reduced thermo-emf, thermal conductivity, and resistivity; $z = \alpha^2/\rho\lambda$; a is the heat transfer coefficient, referred to the unit area of the battery, with the radiator taken into account; $m = a_1/a_2$; j is the current density; W is the water equivalent of flow; $n = W_1/W_2$; Q is the cooling or heating capacity of the thermopile; x is the coordinate along the flow; $\xi = x/l$ is the dimensionless coordinate; $\tau = zT$, is the dimensionless temperature; $\nu = (\alpha d/\lambda)j$ is the dimensionless current; $Bi_{1,2} = 1/\beta_{1,2} = a_{1,2}d/\lambda$, is the Biot number; $R_{1,2} = S\lambda/W_{1,2}d$, $N_{1,2} = Sa_{1,2}/W_{1,2}$ is the dimensionless area; $P_{1,2} = Va_{1,2}^2/W_{1,2}\lambda$ is the dimensionless volume. The subscripts 1 and 2 refer to the cooled and heated media, respectively.

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